Math 3236 Statistical Theory

4/6/23

$X \quad \beta'(\alpha, \beta)$

$$f_X(x) = \frac{x^{\alpha-1}(1+x)^{-\alpha-\beta}}{B(\alpha, \beta)}$$

$U \quad \chi^2 \quad n$ d.o.f.

$V \quad \chi^2 \quad m$ d.o.f.

U/V is $\beta'\left(n/2, m/2\right)$

$Y = V \qquad X = U/V$

$U = XY \qquad V = Y$

Jacobian is y

$$\left|\det\begin{pmatrix} y & x \\ 0 & 1 \end{pmatrix}\right| = y$$

$$f_{XY}(x,y) = \frac{1}{2^{n/2}\Gamma(n/2)}\frac{1}{2^{m/2}\Gamma(m/2)}$$

$$y\, y^{m/2-1}(xy)^{n/2-1} e^{-y(1+x)/2}$$

$$f_X(x) = \int_0^{\infty} f_{XY}(x,y)\,dy =$$

$$\frac{1}{2^{n/2}\Gamma(n/2)}\frac{1}{2^{m/2}\Gamma(m/2)} x^{n/2-1}$$

$$\int_0^{\infty} y^{(n+m)/2-1} e^{-y(1+x)/2}\,dy$$

$$\int_0^\infty x^{\alpha-1} e^{-\beta x}\,dx = \frac{\Gamma(\alpha)}{\beta^\alpha}$$

$$= \frac{1}{2^{n/2}\Gamma(n/2)} \frac{1}{2^{m/2}\Gamma(m/2)} x^{n/2-1} \left(\frac{1+x}{2}\right)^{-(n+m)/2}$$

$$\Gamma\left((n+m)/2\right)$$

$$\_\_\_\_\_\_\_\_\_\_ \circ \_\_\_\_\_\_\_\_\_\_$$

$$Y = \frac{X}{1+X} \quad \text{is} \quad \beta(\alpha, \beta)$$

$$x = \frac{y}{1-y} \qquad \frac{dx}{dy} = \frac{1}{(1-y)^2}$$

$$1+x = \frac{1}{1-y}$$

$$f_Y(y) = \frac{1}{B(\alpha,\beta)} \frac{1}{(1-y)^2} \left(\frac{y}{1-y}\right)^{\alpha-1} (1-y)^{\alpha+\beta} =$$

$$= y^{\alpha-1}(1-y)^{\beta-1} / B(\alpha,\beta)$$

$$X = \frac{U}{V} \qquad Y = \frac{X}{1+X} = \frac{U}{U+V}$$

$X \quad \chi^2_n \qquad Y \quad \chi^2_m$

$$\frac{X}{n} \Big/ \frac{Y}{m} \simeq F(n, m)$$

X_i i.i.d. normal $\mu_X \; \sigma_X^2$

Y_j i.i.d. normal $\mu_Y \; \sigma_Y^2$

$i = 1 \dots N$

$j = 1 \dots M$

$$T = \frac{\sigma_Y^2}{\sigma_X^2} \frac{\sum_{i=1}^{N} (X_i - \overline{X})^2}{\sum_{j=1}^{M} (Y_i - \overline{Y})^2}$$

$$\frac{1}{\sigma_X^2}\sum_i (X_i - \bar{X})^2 = \chi^2_{N-1}$$

$$\frac{1}{\sigma_Y^2}\sum_j (Y_j - \bar{Y})^2 = \chi^2_{M-1}$$

0

$a_1 \qquad \alpha_2 \qquad\qquad \alpha_1 + \alpha_2 = 1 - \gamma$

$$B'_{(N-1)/2,\,(M-1)/2}(b_2) = 1 - \alpha_2$$

$$B'_{(N-1)/2,\,(M-1)/2}(b_1) = \alpha_1$$

$$\mathbb{P}(b_1 \leq T \leq b_2) = \gamma$$

$$\to \sqrt{\quad} \qquad b_1 \leq \frac{\sigma_Y}{\sigma_X} \leq \sqrt{b_2 \frac{\sum_{j=1}^{M} (Y_j - \bar{Y})}{\sum_{i=1}^{N} (X_i - \bar{X})^2}}$$

$X \quad Y$

$$\mathbb{E}(X) = \mathbb{E}(Y) = 0$$

$$Var(X) = Var(Y) = 1$$

$$cov(X, Y) = \rho$$

$U = (X+Y)/\sqrt{2} \qquad V = (X-Y)/\sqrt{2}$

$$\mathbb{E}(U) = \mathbb{E}(V) = 0$$

$$\mathbb{E}(UV) = \mathbb{E}(X^2) - \mathbb{E}(Y^2) = 0$$

$$S = \frac{1-\rho}{1+\rho} \frac{\sum_{i=1}^{N} (U_i - \bar{U})^2}{\sum_{j=1}^{N} (V_j - \bar{V})}$$

$$S \quad \beta'\left(\frac{N-1}{2}, \frac{N-1}{2}\right)$$

$$\mathbb{P}(S \leq b) = \gamma$$

$$S \leq b$$

$$\frac{1-\rho}{1+\rho} \leq b \tilde{A}(X, Y)$$

$$\tilde{A}(X, Y) = \frac{\sum_{i=1}^{N} (V_i - \bar{V})^2}{\sum_{j=1}^{N} (U_j - \bar{U})^2}$$

T_i exp λ

$$\mathbb{E}(T_i) = 1/\lambda$$

$$\bar{T} \simeq N\left(\frac{1}{\lambda}, \frac{1}{N\lambda^2}\right)$$

$$\sqrt{N}\lambda\left(\bar{T} - \frac{1}{\lambda}\right) \simeq N(0,1)$$